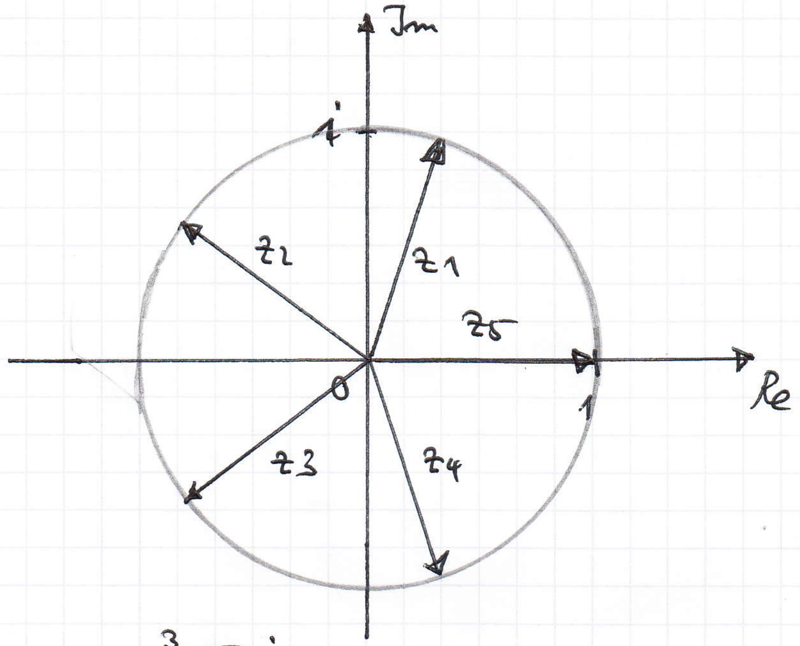
-1- Vertiefungskurs Mathematik Klausur Nr. 4 10.04.19

3 A1 $z_1 = e^{\frac{2}{5}\pi i}$; $z_2 = e^{\frac{4}{5}\pi i}$; $z_3 = e^{\frac{6}{5}\pi i}$; $z_4 = e^{\frac{8}{5}\pi i}$; $z_5 = 1$



2 A2
$$z_1 = 3e^{\frac{3}{10}\pi i}$$
$$z_2 = 3e^{\left(\frac{3}{10}+\frac{5}{10}\right)\pi i} = 3e^{\frac{4}{5}\pi i}$$
$$z_3 = 3e^{\left(\frac{3}{10}+2\cdot\frac{5}{10}\right)\pi i} = 3e^{\frac{13}{10}\pi i}$$
$$z_4 = 3e^{\left(\frac{3}{10}+3\cdot\frac{5}{10}\right)\pi i} = 3e^{\frac{9}{5}\pi i}$$

6 A3 3 a) $z^3 - 6z^2 + 13z = 0$

$z\cdot(z^2 - 6z + 13) = 0$ $\quad z_1 = 0$

$z^2 - 6z + 13 = 0$ $\quad D = 36 - 52 = -16$

$$z_{2;3} = \frac{6 \pm \sqrt{16}\, i}{2} = \frac{6 \pm 4i}{2} = 3 \pm 2i$$

$L = \{0;\ 3+2i;\ 3-2i\}$

3 b) $2z^4 + 4z^2 - 16 = 0$

$z^4 + 2z^2 - 8 = 0$ $\quad$ Sub.: $z^2 = u$

$u^2 + 2u - 8 = 0$

$(u+4)\cdot(u-2) = 0$

$u_1 = -4 \quad u_2 = 2$

$z_1 = 2i \quad z_2 = -2i \quad z_3 = \sqrt{2} \quad z_4 = -\sqrt{2}$

$L = \{-\sqrt{2};\ \sqrt{2};\ 2i;\ -2i\}$

11VP

4 A4 $z_1 = 1-2i \Rightarrow z_2 = 1+2i$

$$(z-(1-2i))\cdot(z-(1+2i)) = z^2-(1-2i+1+2i)z+1+4$$

$$= z^2-2z+5$$

Polynomdivision:

$$(z^4-2z^3+2z^2+6z-15):(z^2-2z+5) = z^2-3$$

$$-\quad \underline{z^4-2z^3+5z^2}$$

$$-3z^2+6z-15$$

$$-\quad \underline{-3z^2+6z-15}$$

$$0$$

$$z^2-3=0 \Rightarrow z^2=3 \Rightarrow \begin{array}{l} z_3=\sqrt{3} \\ z_4=-\sqrt{3} \end{array}$$

3 A5 1 a) $f(x) = x^2\cdot(x^2+1)$

1 b) $g(x) = (x^2+1)^2$

1 c) $h(x) = x^3\cdot(x-1)\cdot(x^2+1)$

3 A6 $\dfrac{3x-9}{x^2-1} = \dfrac{3x-9}{(x-1)\cdot(x+1)} = \dfrac{A}{x-1} + \dfrac{B}{x+1}$

$$\frac{3x-9}{(x-1)\cdot(x+1)} = \frac{Ax+A+Bx-B}{(x-1)\cdot(x+1)} = \frac{(A+B)x+A-B}{(x-1)\cdot(x+1)}$$

$$A+B=3$$
$$A-B=-9 \quad +$$
$$\overline{2A \quad =-6} \Rightarrow A=-3 \Rightarrow B=6$$

$$\frac{3x-9}{x^2-1} = \frac{6}{x+1} - \frac{3}{x-1}$$

8 A7 3,5 a) $\displaystyle\int_{-2}^{0} \underbrace{3x}_{u}\cdot\underbrace{e^{-0,5x}}_{v'}\,dx = J$ $\quad u'=3$ $\quad v=-2e^{-0,5x}$

$$J = \left[-3x\cdot 2e^{-0,5x}\right]_{-2}^{0} - \int_{-2}^{0} -6e^{-0,5x}\,dx$$

$$J = 0 + (-6)\cdot 2e + \left[12e^{-0,5x}\right]_{-2}^{0}$$

$$J = -12e - (12-12e) = -12$$

13,5 VP

A7 b) 4,5

$$\int_0^2 \frac{10x}{\sqrt{25-4x^2}}\,dx = J$$

<u>Variante 1:</u>

inhere Ableitung

$$J = -\frac{10}{8}\cdot\int_0^2 \frac{-8x}{\sqrt{25-4x^2}}\,dx = -\frac{5}{4}\cdot\int_0^2 (-8x)\cdot(25-4x^2)^{-\frac{1}{2}}\,dx$$

$$J = -\frac{5}{4}\cdot\left[2\cdot(25-4x^2)^{\frac{1}{2}}\right]_0^2 = -\frac{5}{4}\cdot\left[2\sqrt{25-4x^2}\right]_0^2$$

$$J = -\frac{5}{4}\left(2\cdot\sqrt{9} - 2\sqrt{25}\right) = -\frac{5}{4}\cdot(6-10) = 5$$

<u>Variante 2:</u> Sub.: $x = \frac{5}{2}\sin u$

$$\frac{dx}{du} = \frac{5}{2}\cos u \Rightarrow dx = \frac{5}{2}\cos u \cdot du$$

Grenze: $x = 0 \Rightarrow u = 0$

$x = 2 \Rightarrow \sin u = 0{,}8 \Rightarrow u = \arcsin 0{,}8$

$$J = \int_0^{\arcsin 0{,}8} \frac{25\sin u}{\sqrt{25-25\sin^2 u}}\cdot\frac{5}{2}\cos u\,du$$

$$J = \int_0^{\arcsin 0{,}8} \frac{125\sin u\cdot\cos u}{2\cdot 5\cdot\underbrace{\sqrt{1-\sin^2 u}}_{\cos u}}\,du = \int_0^{\arcsin 0{,}8} \frac{25}{2}\cdot\sin u\,du$$

$$J = \frac{25}{2}\cdot\int_0^{\arcsin 0{,}8} \sin u\,du = \frac{25}{2}\cdot\left[-\cos u\right]_0^{\arcsin 0{,}8}$$

$$J = \frac{25}{2}\cdot\left(-\cos(\arcsin 0{,}8) + \underbrace{\cos 0}_{1}\right)$$

$$\cos(\arcsin 0{,}8) = \sqrt{1-\sin^2(\arcsin 0{,}8)} = \sqrt{1-0{,}8^2} = 0{,}6$$

$$J = \frac{25}{2}\cdot(-0{,}6+1) = \frac{25}{2}\cdot 0{,}4 = 5$$

4 A8 $J = \int_1^5 \frac{\ln(x^4)}{2x}\,dx$

Sub.: $x = e^u \Rightarrow x^4 = (e^u)^4 = e^{4u}$

$$\frac{dx}{du} = e^u \Rightarrow dx = e^u\cdot du$$

Grenzen: $x = 1 \Rightarrow u = 0$

$x = 5 \Rightarrow u = \ln 5$

$$J = \int_0^{\ln 5} \frac{\ln(e^{4u})}{2\cdot e^u}\cdot e^u\,du$$

$$J = \int_0^{\ln 5} \frac{4u}{2}\,du = \int_0^{\ln 5} 2u\,du$$

$$J = \int_0^{\ln 5} 2u\,du = \left[u^2\right]_0^{\ln 5} = (\ln 5)^2$$

33 VP

8,5 VP