-1- Vertiefungskurs Mathematik Klausur Nr. 3 28.11.18 (AP)

4 A1 a) $a_n = \frac{1}{n}$ b) $b_n = n$ c) $c_n = (-1)^n \cdot \frac{1}{n}$ d) $d_n = (-1)^n$

4 A2 a) $a_7 = q^3 \cdot a_4 \Rightarrow q^3 = \frac{a_7}{a_4} = \frac{-48}{6} = -8 \Rightarrow q = -2$

3

$a_4 = q^3 \cdot a_1 \Rightarrow a_4 = -8 \cdot a_1 \Rightarrow a_1 = \frac{a_4}{-8} = \frac{6}{-8} = -\frac{3}{4}$

$a_{14} = q^7 \cdot a_7 = (-2)^7 \cdot (-48) = 6144$

1 b) $a_n = -\frac{3}{4} \cdot (-2)^{n-1}$

7,5 A3 a) $a_n = \frac{5n^2 - 4n + 7}{2n^2 + 4n - 1} = \frac{5 - \frac{4}{n} + \frac{7}{n^2}}{2 + \frac{4}{n} - \frac{1}{n^2}}$

1

$\lim\limits_{n \to \infty} a_n = \frac{5}{2} = 2{,}5$

2 b) $b_n = \frac{3 \cdot 6^n - 4 \cdot 3^{n+1}}{9 \cdot 6^{n-1} + 2 \cdot 5^n} = \frac{3 \cdot 6^n - 12 \cdot 3^n}{1{,}5 \cdot 6^n + 2 \cdot 5^n}$

$b_n = \frac{3 - 12\left(\frac{3}{6}\right)^n}{1{,}5 + 2 \cdot \left(\frac{5}{6}\right)^n}$

$\lim\limits_{n \to \infty} b_n = \frac{3}{1{,}5} = 2$

1,5 c) $c_n = \frac{2n^4 - 3n + 7}{8n^3 + 12n^2 - 1} = \frac{2n - \frac{3}{n^2} + \frac{7}{n^3}}{8 + \frac{12}{n} - \frac{1}{n^3}}$

$c_n \to \infty$ für $n \to \infty$ Also kein Grenzwert

3 d) $d_n = \sqrt{4n^2 + 2n} - \sqrt{4n^2 - 6n + 1}$

$d_n = \frac{\left(\sqrt{4n^2 + 2n} - \sqrt{4n^2 - 6n + 1}\right) \cdot \left(\sqrt{4n^2 + 2n} + \sqrt{4n^2 - 6n + 1}\right)}{\sqrt{4n^2 + 2n} + \sqrt{4n^2 - 6n + 1}}$

$d_n = \frac{4n^2 + 2n - (4n^2 - 6n + 1)}{\sqrt{4n^2 + 2n} + \sqrt{4n^2 - 6n + 1}} = \frac{8n - 1}{\sqrt{4n^2 + 2n} + \sqrt{4n^2 - 6n + 1}}$

$d_n = \frac{8 - \frac{1}{n}}{\sqrt{4 + \frac{2}{n}} + \sqrt{4 - \frac{6}{n} + \frac{1}{n^2}}}$ $\lim\limits_{n \to \infty} d_n = \lim\limits_{n \to \infty} \frac{8}{2+2} = 2$

15,5 VP

5,5 A4, 1 a) $(3+2i)+(-2+3i) = 1+5i$

1 b) $(5-3i)-(-3-5i) = 5-3i+3+5i = 8+2i$

1,5 c) $(2+3i)\cdot(2-i) = 4+3+6i-2i = 7+4i$

2 d) $\frac{6-3i}{3-4i} = \frac{(6-3i)\cdot(3+4i)}{(3-4i)\cdot(3+4i)} = \frac{18+12-9i+24i}{9+16} = \frac{30+15i}{25}$

$= \frac{6+3i}{5} = \frac{6}{5} + \frac{3}{5}i$

5 A5, 2 a) $\overline{(2-3i)}\cdot(2-3i)^2 = \underbrace{\overline{(2-3i)}}_{2+3i}\cdot(2-3i)\cdot(2-3i)$

$= (4+9)\cdot(2-3i) = 13\cdot(2-3i) = 26-39i$

1,5 b) $2\cdot e^{\frac{5}{3}\pi i}\cdot 3e^{\frac{3}{2}\pi i} = 6\cdot e^{(\frac{5}{3}+\frac{3}{2})\pi i} = 6\cdot e^{\frac{19}{6}\pi i} = 6e^{\frac{7}{6}\pi i}$

1,5 c) $8e^{\frac{2}{5}\pi i} : 4e^{\frac{13}{10}\pi i} = 2\cdot e^{(\frac{2}{5}-\frac{13}{10})\pi i} = 2\cdot e^{-\frac{9}{10}\pi i} = 2\cdot e^{\frac{11}{10}\pi i}$

4 A6, 1,5 a) $(\sqrt{3}e^{0,7\pi i})^8 = \sqrt{3}^8\cdot e^{8\cdot 0,7\pi i} = 81e^{5,6\pi i} = 81\cdot e^{1,6\pi i}$

2,5 b) $(1-i)^9 = (\sqrt{2}\cdot e^{-\frac{\pi}{4}i})^9 = \sqrt{2}^9\cdot e^{-\frac{9}{4}\pi i} = 16\sqrt{2}\cdot e^{\frac{7}{4}\pi i}$

2 A7 $z = 2-2\sqrt{3}i$ $\quad r^2 = 2^2+(2\sqrt{3})^2 = 4+12 = 16 \Rightarrow r = 4$

$\tan\varphi = \frac{-2\sqrt{3}}{+2} = -\sqrt{3} \Rightarrow \varphi = -\frac{1}{3}\pi \mathrel{\hat{=}} \frac{5}{3}\pi$

$z = 4e^{\frac{5}{3}\pi i}$

4 A8 Sei $\varepsilon > 0$

z.z. $|a_n - g| < \varepsilon$ für alle $n \geq N_\varepsilon$

$\left|\frac{8n-3}{2n+4} - 4\right| = \left|\frac{8n-3}{2n+4} - \frac{8n+16}{2n+4}\right| = \left|-\frac{19}{2n+4}\right| = \frac{19}{2n+4}$

$\frac{19}{2n+4} < \varepsilon \quad |\cdot(2n+4)$

$19 < \varepsilon\cdot(2n+4) \quad |:\varepsilon \quad (\varepsilon>0)$

$\frac{19}{\varepsilon} < 2n+4 \quad |-4$

$\frac{19}{\varepsilon} - 4 < 2n \quad |:2$

$\frac{19}{2\varepsilon} - 2 < n$

N_ε ist also die kleinste natürliche Zahl ~~die nach~~ mit $N_\varepsilon > \frac{19}{2\varepsilon} - 2$.

20,5 VP

3 A9 1. Bedingung $24 - 4a_1 \geqq 0$

$$\Rightarrow a_1 \leqq 6$$

2. Bedingung $24 - 4a_2 \geqq 0$

$$24 - 4 \cdot \sqrt{24-4a_1} \geqq 0$$

$$\sqrt{24-4a_1} \leqq 6$$

$$24 - 4a_1 \leqq 36 \quad | + 4a_1$$

$$-12 \leqq 4a_1 \quad | : 4$$

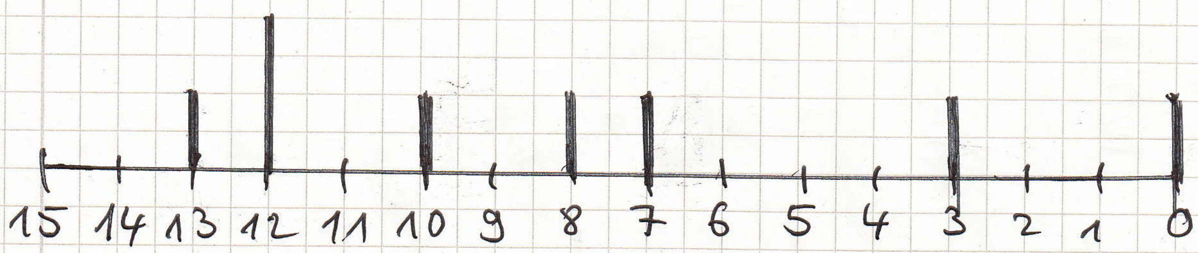
$$-3 \leqq a_1$$

Also muss gelten $-3 \leqq a_1 \leqq 6$, damit es unendlich viele Folgeglieder gibt.

39VP

Notenspiegel VK M 12 Klausur 1 28.11.18 (AP)

Durchschnitt: 8,1 NP 1 Schüler $\hat{=}$ 1cm



3VP