5,5 A1 $_1$ a) $1+5i$ $_1$ b) $10+4i$

1,5 c) $(4+3i)\cdot(1-2i) = 4-6i^2-8i+3i = 4+6-5i = 10-5i$

2 d) $\dfrac{3-2i}{2+i} = \dfrac{(3-2i)\cdot(2-i)}{(2+i)\cdot(2-i)} = \dfrac{6+2i^2-3i-4i}{5} = \dfrac{4-7i}{5} = \dfrac{4}{5}-\dfrac{7}{5}i$

3,5 A2 $_{1,5}$ a) $\underbrace{(1+2i)\cdot(1-2i)}_{5}\cdot(1-2i) = 5\cdot(1-2i) = 5-10i$

1 b) $12e^{(\frac{4}{3}+\frac{3}{2})\pi i} = 12e^{\frac{17}{6}\pi i} = 12e^{\frac{5}{6}\pi i}$

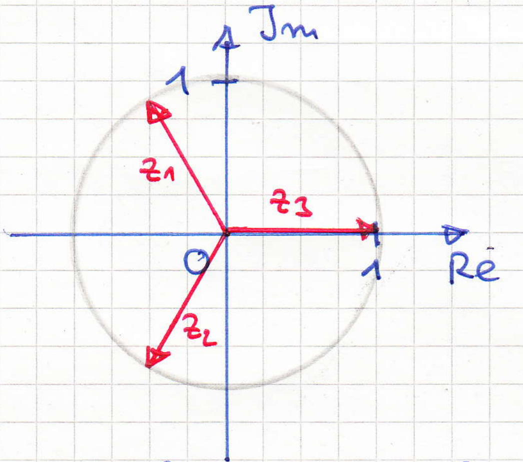
1 c) $3e^{(\frac{3}{5}-\frac{11}{10})\pi i} = 3e^{-\frac{1}{2}\pi i} = 3e^{\frac{3}{2}\pi i}$

3 A3 $_1$ a) $\sqrt{2}^{\,6}\, e^{4{,}2\pi i} = 8e^{0{,}2\pi i}$

2 b) $(1+i)^{10} = \left(\sqrt{2}\,e^{\frac{\pi}{4}i}\right)^{10} = \sqrt{2}^{\,10}\, e^{\frac{10}{4}\pi i} = 32e^{\frac{1}{2}\pi i} = 32i$

Alternativ: $(1+i)^{10} = \left((1+i)^2\right)^5 = (1+i^2+2i)^5 = (2i)^5 = 32i^5 = 32i$

3 A4



$z_1 = e^{\frac{2\pi}{3}i}$

$z_2 = e^{\frac{4\pi}{3}i}$

$z_3 = 1$

2,5 A5 $_1$ $z_1 = 2e^{\frac{6}{20}\pi i} = 2e^{\frac{3}{10}\pi i}$

0,5 $z_2 = 2e^{(\frac{3}{10}\pi+\frac{2\pi}{4})i} = 2e^{\frac{4}{5}\pi i}$

0,5 $z_3 = 2e^{(\frac{3}{10}\pi+2\cdot\frac{2\pi}{4})i} = 2e^{\frac{13}{10}\pi i}$

0,5 $z_4 = 2e^{(\frac{3}{10}\pi+3\cdot\frac{2\pi}{4})i} = 2e^{\frac{9}{5}\pi i}$

4 A6 $_2$ a) $r = \sqrt{a^2+b^2} = \sqrt{1^2+\sqrt{3}^2} = \sqrt{4} = 2$

$\tan\varphi = \dfrac{b}{a} = \dfrac{\sqrt{3}}{1} = \sqrt{3} \Rightarrow \varphi = \dfrac{\pi}{3}$ $\qquad z = 2e^{\frac{\pi}{3}i}$

2 b) $a = r\cdot\cos\varphi = \sqrt{18}\cdot\cos(\frac{7}{4}\pi) = \sqrt{18}\cdot\frac{1}{2}\sqrt{2} = 3$

$b = r\cdot\sin\varphi = \sqrt{18}\cdot\sin(\frac{7}{4}\pi) = \sqrt{18}\cdot(-\frac{1}{2}\sqrt{2}) = -3$

$z = 3-3i$

24,5 VP

6,5 A7 a) 3 $z^3 - 4z^2 + 13z = z\cdot(z^2 - 4z + 13) = 0$

$z_1 = 0$ oder $z^2 - 4z + 13 = 0$

$D = 16 - 52 = -36$

$z_{2;3} = \dfrac{4 \pm \sqrt{-36}}{2} = \dfrac{4 \pm 6i}{2} = 2 \pm 3i$ $z_2 = 2+3i$ $z_3 = 2-3i$

3,5 b) $z^4 + z^2 - 12 = 0$ Sub.: $z^2 = u$

$u^2 + u - 12 = 0$

$(u+4)\cdot(u-3) = 0 \Rightarrow u_1 = -4$; $u_2 = 3$

Resub.: $-4 = z^2 \Rightarrow z_1 = 2i$; $z_2 = -2i$

$3 = z^2 \Rightarrow z_3 = \sqrt{3}$; $z_4 = -\sqrt{3}$

3 A8 $\dfrac{3x+7}{x^2-1} = \dfrac{a}{x-1} + \dfrac{b}{x+1} = \dfrac{a\cdot(x+1) + b(x-1)}{(x-1)\cdot(x+1)}$

$= \dfrac{(a+b)x + a - b}{x^2-1}$

$\Rightarrow \left.\begin{array}{l} a+b = 3 \\ a-b = 7 \end{array}\right\} 2a = 10 \Rightarrow a = 5 \Rightarrow b = -2$

$\dfrac{3x+7}{x^2-1} = \dfrac{5}{x-1} - \dfrac{2}{x+1}$

8 A9 a) 4 $J = \int_0^2 2x e^{-0,5x} dx$ $u(x) = 2x \Rightarrow u'(x) = 2$

$v'(x) = e^{-0,5x} \Rightarrow v(x) = -2e^{-0,5x}$

$J = \left[2x(-2)\cdot e^{-0,5x}\right]_0^2 - \int_0^2 -4e^{-0,5x} dx$

$J = -8e^{-1} - 0 - \left[8e^{-0,5x}\right]_0^2 = -8e^{-1} - (8e^{-1} - 8)$

$J = -16e^{-1} + 8 = 8 - \dfrac{16}{e}$

4 b) $J = \int_0^4 \dfrac{6x}{\sqrt{25-x^2}} dx = (-3)\cdot\int_0^4 \dfrac{-2x}{(25-x^2)^{\frac{1}{2}}} dx = (-3)\int_0^4 (-2x)\cdot(25-x^2)^{-\frac{1}{2}} dx$

(Beachte: Im Zähler steht die <u>innere</u> Ableitung des Nenners)

$J = (-3)\cdot\left[2(25-x^2)^{\frac{1}{2}}\right]_0^4 = (-6)\cdot\left[\sqrt{25-x^2}\right]_0^4 = (-6)\cdot(3-5)$

$J = (-6)\cdot(-2) = 12$

17,5 VP

A9 b) Alternativ: Substitution: $x = 5 \cdot \sin u$

$$\frac{dx}{du} = 5 \cdot \cos u \Rightarrow dx = 5\cos u \cdot du$$

$x = 0 \Rightarrow u = 0$; $x = 4 \Rightarrow u = \arcsin 0{,}8$

$$J = \int_0^{\arcsin 0{,}8} \frac{30 \sin u}{\sqrt{25 - 25\sin^2 u}} \cdot 5\cos u\, du = \int_0^{\arcsin 0{,}8} \frac{30 \cdot \sin u}{5 \cos u} \cdot 5\cos u \cdot du$$

$$J = \int_0^{\arcsin 0{,}8} 30 \sin u\, du = \left[-30 \cos u\right]_0^{\arcsin 0{,}8}$$

$$\cos(\arcsin 0{,}8) = \sqrt{1 - \sin^2(\arcsin 0{,}8)} = \sqrt{1 - 0{,}8^2} = 0{,}6$$

$$J = -30 \cdot 0{,}6 - (-30) = -18 + 30 = 12$$

4 A10 $J = \int_1^3 \frac{\ln(x^2)}{x}\, dx$ Sub.: $x = e^u \Rightarrow x^2 = (e^u)^2 = e^{2u}$

$$\frac{dx}{du} = e^u \Rightarrow dx = e^u \cdot du$$

$1 = e^u \Rightarrow u = 0$ $3 = e^u \Rightarrow u = \ln 3$

$$J = \int_0^{\ln 3} \frac{\ln(e^{2u})}{e^u} \cdot e^u du = \int_0^{\ln 3} 2u\, du = \left[u^2\right]_0^{\ln 3} = (\ln 3)^2$$

5 A11 Weg 1: Viertelkreis: $x(t) = 2\cos t$ $x'(t) = -2\sin t$

$0 \leq t \leq \frac{\pi}{2}$ $y(t) = 2\sin t$ $y'(t) = 2\cos t$

$$\sqrt{x'(t)^2 + y'(t)^2} = \sqrt{4\sin^2 t + 4\cos^2 t} = \sqrt{4} = 2 \text{ Linienelement}$$

$$f(t) = 3 \cdot 2\cos t \cdot (1 - 4\sin^2 t) = 6\cos t - 24 \cos t \cdot \sin^2 t$$

$$J_1 = \int_0^{\frac{\pi}{2}} (6\cos t - 24 \cos t \cdot \sin^2 t) \cdot 2\, dt = 2 \cdot \int_0^{\frac{\pi}{2}} (6\cos t - 24 \cos t \sin^2 t)\, dt$$

$$J_1 = 2 \cdot \left[6 \sin t - 8 \sin^3 t\right]_0^{\frac{\pi}{2}} = 2 \cdot (6 - 8 - 0) = 2 \cdot (-2) = -4$$

Weg 2: $y(x) = 2 - x$ $0 \leq x \leq 2$

$y'(x) = -1$ Linienelement: $\sqrt{1 + y'(x)^2} = \sqrt{1+1} = \sqrt{2}$

$$f(x) = 3x \cdot (1 - (2-x)^2) = 3x \cdot (1 - 4 + 4x - x^2) = 3x \cdot (-3 + 4x - x^2)$$

$$f(x) = -3x^3 + 12x^2 - 9x$$

$$J_2 = \int_0^2 (-3x^3 + 12x^2 - 9x) \cdot \sqrt{2}\, dx = \sqrt{2} \cdot \left[-\frac{3}{4}x^4 + 4x^3 - \frac{9}{2}x^2\right]_0^2$$

$$J_2 = \sqrt{2} \cdot (-12 + 32 - 18 - 0) = 2\sqrt{2}$$

9 VP

Σ 48 VP