7 A1 3 a) $z^3 - 6z^2 + 13z = 0$

$z \cdot (z^2 - 6z + 13) = 0 \qquad z_1 = 0$

$z^2 - 6z + 13 = 0 \qquad D = (-6)^2 - 4 \cdot 13 = 36 - 52 = -16$

$z_{2;3} = \dfrac{6 \pm \sqrt{-16}}{2} = \dfrac{6 \pm 4i}{2} = 3 \pm 2i$

$z_2 = 3 + 2i \qquad z_3 = 3 - 2i$

4 b) $z^4 - 8z^2 - 9 = 0$

Sub.: $z^2 = u$

$u^2 - 8u - 9 = 0$

Vieta: $(u-9)\cdot(u+1) = 0 \qquad u_1 = 9 \qquad u_2 = -1$

Resub: $z^2 = 9 \qquad z_1 = 3 \qquad z_2 = -3$

$z^2 = -1 \qquad z_3 = i \qquad z_4 = -i$

4 A2 $z_1 = 1 + 2i \Rightarrow z_2 = \overline{z_1} = 1 - 2i$

$(z - z_1)\cdot(z - z_2) = (z - (1+2i))\cdot(z - (1-2i))$

$= z^2 - 2z + 5$

Polynomdivision: $(z^4 - 2z^3 + 3z^2 + 4z - 10) : (z^2 - 2z + 5) = z^2 - 2$

$-\ (z^4 - 2z^3 + 5z^2)$

$-2z^2 + 4z - 10$

$-\ (-2z^2 + 4z - 10)$

0

$z^2 - 2 = 0$

$z^2 = 2 \Rightarrow z_3 = \sqrt{2} \ ; \ z_4 = -\sqrt{2}$

3 A3 a) z.B $f(x) = x \cdot (x-1) \cdot (x^2+1)$

b) z.B $g(x) = (x^2+1)^2$

c) z.B $h(x) = x^2 \cdot (x^2+1) \cdot (x^2+4)$

3 A4 $\dfrac{2x+6}{x^2-1} = \dfrac{2x+6}{(x-1)\cdot(x+1)} = \dfrac{A}{x-1} + \dfrac{B}{x+1}$

$A(x+1) + B\cdot(x-1) = \underbrace{(A+B)}_{2}x + \underbrace{A-B}_{6}$

$A + B = 2$
$A - B = 6 \quad | +$
$2A = 8 \qquad A = 4$

$B = 2 - A = 2 - 4 = -2$

(17VP)

A4 $\frac{2x+6}{x^2-1} = \frac{4}{x-1} + \frac{-2}{x+1} = \frac{4}{x-1} - \frac{2}{x+1}$

14 A5 4 a) $\int_0^1 \underbrace{4x}_{u} \cdot \underbrace{e^{-0{,}5x}}_{v'} dx = J \qquad u' = 4 \quad v = -2e^{-0{,}5x}$

$J = \left[-8x e^{-0{,}5x}\right]_0^1 - \int_0^1 -8e^{-0{,}5x} dx$

$J = -8e^{-0{,}5} + \left[-16e^{-0{,}5x}\right]_0^1$

$J = -\frac{8}{\sqrt{e}} - \frac{16}{\sqrt{e}} - (-16) = 16 - \frac{24}{\sqrt{e}}$

5 b) $\frac{15}{(x-1)\cdot(x+2)} = \frac{A}{x-1} + \frac{B}{x+2}$

$A\cdot(x+2) + B(x-1) = 15$

$\underbrace{(A+B)}_{0} x + \underbrace{2A-B}_{15} = 15$ für alle $x \in \mathbb{R}\setminus\{1;-2\}$

$\begin{array}{l} A+B=0 \\ 2A-B=15 \end{array} \Big| + \Rightarrow 3A = 15 \Rightarrow A = 5 \Rightarrow B = -5$

$\int_2^3 \frac{15}{(x-1)\cdot(x+2)} dx = \int_2^3 \frac{5}{x-1} dx - \int_2^3 \frac{5}{x+2} dx$

$= \left[5\ln(x-1)\right]_2^3 - \left[5\ln(x+2)\right]_2^3$

$= 5\ln 2 - 5\underbrace{\ln 1}_{0} - (5\ln 5 - 5\ln 4)$

$= 5\cdot(\ln 2 - \ln 5 + \ln 4) = 5\cdot\ln\frac{8}{5}$

Innere Ableitung der Wurzel

5 c) $\int_0^3 \frac{4x}{\sqrt{25-x^2}} dx = \int_0^3 (-2)\cdot\frac{-2x}{\sqrt{25-x^2}} dx = -2\cdot\int_0^3 \frac{-2x}{\sqrt{25-x^2}} dx$

$= -2\left[2\cdot(25-x^2)^{\frac{1}{2}}\right]_0^3 = -2\cdot\left(2\cdot\sqrt{16} - 2\cdot\sqrt{25}\right)$

$= -2\cdot(8-10) = -2\cdot(-2) = 4$

5 A6 Substitution: $x = e^u \Rightarrow \frac{dx}{du} = e^u \Rightarrow dx = e^u\cdot du$

Neue Grenze: $e^u = 1 \Rightarrow u_1 = 0$; $e^u = 2 \Rightarrow u_2 = \ln 2$

$\int_1^2 \frac{\ln(x^2)}{x} dx = \int_0^{\ln 2} \frac{\ln((e^u)^2)}{e^u}\cdot e^u du = \int_0^{\ln 2} \ln(e^{2u}) du$

$= \int_0^{\ln 2} 2u\, du = \left[u^2\right]_0^{\ln 2} = (\ln 2)^2$

19 VP

4 A7 1 a) $A^T = \begin{pmatrix} 2 & 0 & 1 & -2 \\ -1 & 1 & 2 & 2 \\ 0 & 3 & -3 & 1 \end{pmatrix}$

2 b) $A \cdot B = \begin{pmatrix} 2 & -1 & 0 \\ 0 & 1 & 3 \\ 1 & 2 & -3 \\ -2 & 2 & 1 \end{pmatrix} \cdot \begin{pmatrix} 3 & 1 \\ -1 & 2 \\ 0 & 3 \end{pmatrix} = \begin{pmatrix} 7 & 0 \\ -1 & 11 \\ 1 & -4 \\ -8 & 5 \end{pmatrix}$

1 c) Da B zwei Spalten und A vier Zeilen hat.
Es müsste jedoch die gleiche Anzahl sein, damit die Multiplikation definiert ist.

2 A8

$$\begin{array}{ccc} 1 & 4 & 0 \\ -2 & 0 & -4 \\ 3 & -1 & 2 \\ 1 & 4 & 0 \\ -2 & 0 & -4 \end{array}$$

$\det(C) = 0 + 0 + (-48) - 0 - 4 - (-16)$

$\det(C) = -36$

4 A9 Ansatz $\begin{pmatrix} -1 & 1 \\ 4 & 2 \end{pmatrix} \cdot \begin{pmatrix} a & b \\ c & d \end{pmatrix} = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$

$\Rightarrow \begin{array}{l} -a + c = 1 \\ 4a + 2c = 0 \end{array} \left| \begin{array}{l} \cdot 4 \\ + \end{array} \right.$ bzw. $\begin{array}{l} -b + d = 0 \\ 4b + 2d = 1 \end{array} \left| \begin{array}{l} \cdot 4 \\ + \end{array} \right.$

$6c = 4$ $\qquad$ $6d = 1$

$\Rightarrow c = \frac{2}{3} \Rightarrow a = -\frac{1}{3}$ $\qquad$ $\Rightarrow d = \frac{1}{6} \Rightarrow b = \frac{1}{6}$

$D^{-1} = \begin{pmatrix} -\frac{1}{3} & \frac{1}{6} \\ \frac{2}{3} & \frac{1}{6} \end{pmatrix}$

8VP

Σ 46VP

Alternative Lösung zu 5c)

Substitution: $x = 5 \sin u \Rightarrow \frac{dx}{du} = 5 \cos u \Rightarrow dx = 5 \cos u \cdot du$

Neue Grenze: $5 \sin u = 0 \Rightarrow u_1 = 0$; $5 \sin u = 3 \Rightarrow u_2 = \arcsin \frac{3}{5}$

$$\int_0^3 \frac{4x}{\sqrt{25 - x^2}} dx = \int_0^{\arcsin 0{,}6} \frac{4 \cdot 5 \sin u}{\sqrt{25 - 25 \sin^2 u}} \cdot 5 \cos u \, du$$

$$= \int_0^{\arcsin 0{,}6} \frac{20 \sin u \cdot 5 \cos u}{\sqrt{25 \cdot \underbrace{(1 - \sin^2 u)}_{\cos^2 u}}} du = \int_0^{\arcsin 0{,}6} \frac{20 \sin u \cdot 5 \cos u}{5 \cdot \cos u} du = \int_0^{\arcsin 0{,}6} 20 \sin u \, du$$

$$= \Big[-20 \cos u \Big]_0^{\arcsin 0{,}6} = -20 \cdot \left(\cos(\arcsin 0{,}6) - 1 \right)$$

$$= -20 \cdot \left(\sqrt{1 - \sin^2(\arcsin 0{,}6)} - 1 \right) = -20 \cdot \left(\sqrt{1 - 0{,}6^2} - 1 \right)$$

$$= -20 \left(\sqrt{0{,}64} - 1 \right) = -20 \cdot (0{,}8 - 1) = -20 \cdot (-0{,}2) = 4$$