Klausur 18-20,3

Donnerstag, 16. Januar 2020 21:40

Punkte

1) $z_1 = (1+i)^2 = 1 + 2i + i^2 = 2i$ 0,5

$z_2 = \left(2e^{i\cdot\frac{\pi}{4}}\right)^4 = 8e^{i\pi} = -8$ 0,5

$z_3 = (5-2i)(5+2i) = 25-(2i)^2 = 29$ 1

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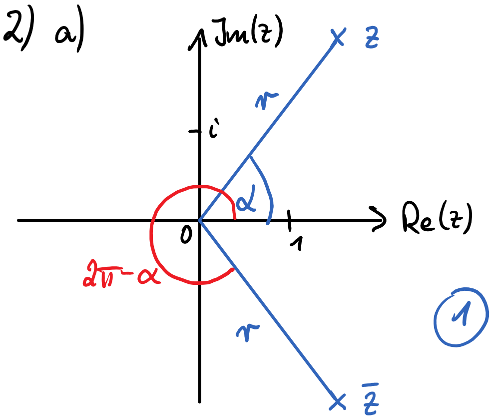
2) a)



1

b) $z = r \cdot e^{i\cdot\alpha}$

$\bar{z} = r\cdot e^{i\cdot(2\pi-\alpha)}$

0,5

z und $\bar{z}$ haben den gleichen Betrag r. Ihre Argumente ergänzen sich zu 2π.

1

/2,5

3. $z_1 = -2+5i$; $z_2 = 3-i$

a) $z_1 + z_2 = 1+4i$ 0,5

b) $z_1 - z_2 = -5+6i$ 0,5

c) $z_1 \cdot z_2 = (-2+5i)(3-i) = -6+2i+15i-5i^2 = -1+17i$ 1

d) $\frac{z_1}{z_2} = \frac{(-2+5i)(3+i)}{(3-i)(3+i)} = \frac{-6-2i+15i+5i^2}{3+1} = \frac{1}{4}(-11+13i) = -\frac{11}{4}+\frac{13}{4}i$ 1,5

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4. $z^4 + 3z^2 - 4 \stackrel{!}{=} 0 \qquad z \in \mathbb{C}$

Subst: $z^2 = u$

$u^2 + 3u - 4 = 0$

$u_{1,2} = \frac{-3 \pm \sqrt{9+16}}{2} = \frac{-3\pm 5}{2}$ 1

$u_1 = 1$; $u_2 = -4$

Rücksubst: 1) $z^2 = 1$

$z_{1/2} = \pm 1$

2) $z^2 = -4$

$z_{3/4} = \pm 2i$ 1

Linearfaktordarstellung: $(z-1)(z+1)(z-2i)(z+2i)$ 0,5

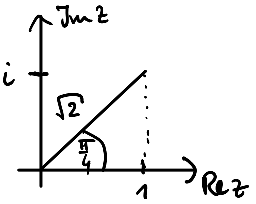
/2,5

5. $z_1 = -2 = 2 \cdot (\cos\pi + i \sin\pi) = 2e^{i\pi}$ 1

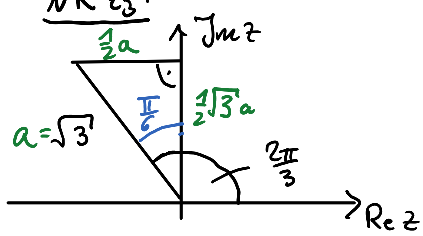
$z_2 = \sqrt{2} \cdot e^{i \cdot \frac{\pi}{4}} = \sqrt{2}\,(\cos\frac{\pi}{4} + i \sin\frac{\pi}{4}) = 1+i$ 1,5

$z_3 = \sqrt{3}\,(\cos\frac{2\pi}{3} + i \sin\frac{2\pi}{3}) = \sqrt{3}\,e^{i \cdot \frac{2\pi}{3}} = -\frac{1}{2}\sqrt{3} + \frac{3}{2}i$ 2,5

NR z_2:



NR z_3:



$a = \sqrt{3}$

$\frac{1}{2}a = \frac{1}{2}\sqrt{3}$

$\frac{1}{2}\sqrt{3}\,a = \frac{1}{2}\sqrt{3} \cdot \sqrt{3} = \frac{3}{2}$

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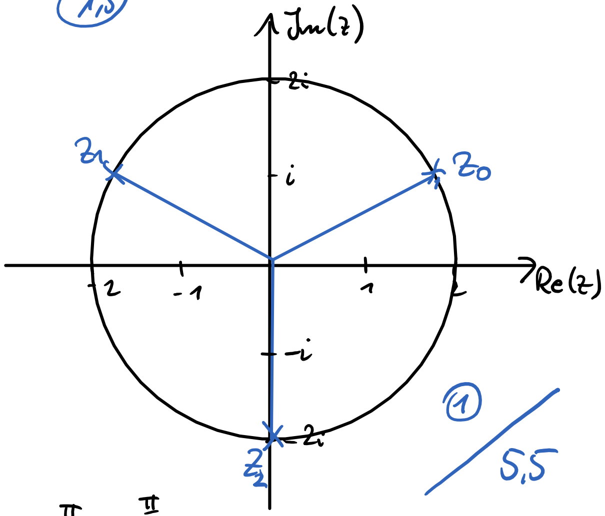
6a) $z_1 = e^{i \cdot \frac{2\pi}{3}}$; $z_2 = e^{i \cdot \frac{4\pi}{3}}$; $z_3 = 1$ 1,5

b) $z^3 = 8i = 8 \cdot e^{i \cdot \frac{\pi}{2}}$ 0,5

$z_0 = 2 \cdot e^{i \cdot \frac{\pi}{6}}$ 1

$z_1 = 2 \cdot e^{i \cdot (\frac{\pi}{6} + \frac{2\pi}{3})} = 2 \cdot e^{i \cdot \frac{5\pi}{6}}$ 1

$z_2 = 2 \cdot e^{i \cdot (\frac{\pi}{6} + \frac{4\pi}{3})} = 2 \cdot e^{i \cdot \frac{3\pi}{2}}$ 0,5



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7. $\int_0^{\frac{\pi}{2}} \underbrace{x}_{u} \cdot \underbrace{\sin(x - \frac{\pi}{2})}_{v'}\,dx = \left[x \cdot (-\cos(x - \frac{\pi}{2}))\right]_0^{\frac{\pi}{2}} - \int_0^{\frac{\pi}{2}} 1 \cdot (-\cos(x - \frac{\pi}{2}))\,dx$

$= -\frac{\pi}{2} \cdot \cos(0) + 0 + \left[\sin(x - \frac{\pi}{2})\right]_0^{\frac{\pi}{2}}$

$= -\frac{\pi}{2} + \sin(0) - \sin(-\frac{\pi}{2})$

$= -\frac{\pi}{2} + 1$

/3

8. $f(x) = x \ln(x^2)$

$\int \underbrace{x}_{v'} \cdot \underbrace{\ln(x^2)}_{u}\,dx = \left[\frac{1}{2}x^2 \ln(x^2)\right] - \int \frac{1}{2}x^2 \cdot \frac{1}{x^2} \cdot 2x\,dx$

$= \left[\frac{1}{2}x^2 \ln(x^2)\right] - \int x\,dx$

$= \left[\frac{1}{2}x^2 \ln(x^2) - \frac{1}{2}x^2\right]$

$F(x) = \frac{1}{2}x^2 \ln(x^2) - \frac{1}{2}x^2$

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$$9.\quad \int_0^{\frac{\pi}{2}} \underbrace{e^{2x}}_{v} \underbrace{\cos(x)}_{u'}\,dx = \left[e^{2x}\sin(x)\right]_0^{\frac{\pi}{2}} - \int_0^{\frac{\pi}{2}} \underbrace{2e^{2x}}_{v} \underbrace{\sin(x)}_{u'}\,dx$$

$$= e^{\pi}\cdot 1 - e^{0}\cdot 0 - \left(\left[-2e^{2x}\cos(x)\right]_0^{\frac{\pi}{2}} - \int_0^{\frac{\pi}{2}} 4e^{2x}\cdot(-\cos(x))\,dx\right)$$

$$= e^{\pi} - \left(\left(-2e^{\pi}\cdot 0 + 2e^{0}\cdot 1\right) + 4\int_0^{\frac{\pi}{2}} e^{2x}\cos(x)\,dx\right)$$

$$= e^{\pi} - 2 - 4\cdot\int_0^{\frac{\pi}{2}} e^{2x}\cos(x)\,dx$$

3

Setze $\int_0^{\frac{\pi}{2}} e^{2x}\cos(x)\,dx = A$.

1

$$A = e^{\pi} - 2 - 4A \qquad | + 4A \quad | : 5$$

$$A = \frac{1}{5}\left(e^{\pi} - 2\right)$$

0,5

4,5

Summe: 31