 Vertiefungskurs Mathematik Klausur Nr. 3 16.01.20 (AP)

5 A1 0,5 a) $(4+3i)+(-2+i) = 2+4i$

1 b) $(5-3i)-(-4+2i) = 9-5i$

1,5 c) $(2-5i)\cdot(-3+2i) = -6+10+15i+4i = 4+19i$

2 d) $\frac{4-2i}{3-4i} = \frac{(4-2i)\cdot(3+4i)}{(3-4i)\cdot(3+4i)} = \frac{12+8+16i-6i}{9+16} = \frac{20+10i}{25}$

$= 0{,}8 + 0{,}4i$

4 A2 2 a) $r = \sqrt{a^2+b^2} = \sqrt{2^2+(2\sqrt{3})^2} = \sqrt{4+12} = \sqrt{16} = 4$

$\tan\varphi = \frac{-2\sqrt{3}}{2} = -\sqrt{3} \Rightarrow \varphi = -\frac{\pi}{3} \hat{=} \frac{5\pi}{3}$

$z = 4\cdot e^{\frac{5\pi}{3}\cdot i}$

2 b) $a = r\cdot\cos\varphi = \cancel{4}^{2}\cdot\cos\frac{3\pi}{4} = -\cancel{2}\sqrt{2}$

$b = r\cdot\sin\varphi = \cancel{4}^{2}\sin\frac{3\pi}{4} = \cancel{2}\sqrt{2}$

$z = -\cancel{2}\sqrt{2} + \cancel{2}\sqrt{2}\cdot i$

5 A3 2 a) $\overline{(3-4i)}\cdot(3-4i)^2 = \underbrace{(3+4i)\cdot(3-4i)}_{9+16}\cdot(3-4i) = 25\cdot(3-4i)$

$= 75-100i$

1,5 b) $3e^{\frac{4}{3}\pi i}\cdot 4e^{\frac{3}{2}\pi i} = 12\cdot e^{\left(\frac{4}{3}+\frac{3}{2}\right)\pi i} = 12\cdot e^{\frac{17}{6}\pi i} = 12\cdot e^{\frac{5}{6}\pi i}$

1,5 c) $8e^{\frac{6}{5}\pi i} : 4e^{\frac{3}{10}\pi i} = 2\cdot e^{\left(\frac{6}{5}-\frac{3}{10}\right)\pi i} = 2\cdot e^{\frac{9}{10}\pi i}$

4 A4 1,5 a) $(\sqrt{3}e^{0{,}8\pi i})^6 = \sqrt{3}^6\cdot e^{6\cdot 0{,}8\pi i} = 27\cdot e^{4{,}8\pi i} = 27\cdot e^{0{,}8\pi i}$

2,5 b) $1-i = \sqrt{2}\cdot e^{-\frac{\pi}{4}i}$

$(1-i)^{11} = (\sqrt{2}\cdot e^{-\frac{\pi}{4}i})^{11} = \sqrt{2}^{11}\cdot e^{-\frac{11\pi}{4}i} = 32\cdot\sqrt{2}\, e^{\frac{5}{4}\pi i}$

A5 3 a) $z^3 - 6z^2 + 13z = 0$

$z\cdot(z^2-6z+13) = 0 \qquad z_1 = 0 \qquad L = \{0; 3+2i; 3-2i\}$

oder $z^2-6z+13 = 0 \quad D = 36-4\cdot 13 = -16$

$z_{2;3} = \frac{6\pm\sqrt{-16}}{2} = \frac{6\pm 4i}{2} = 3\pm 2i \qquad z_2 = 3+2i \quad z_3 = 3-2i$

21VP

A5 b) 4 $2z^4+4z^2-16=0 \quad |:2$

$$z^4+2z^2-8=0$$

Sub.: $z^2=u$

$$u^2+2u-8=0$$

Vieta: $(u+4)\cdot(u-2)=0 \qquad u_1=-4 \quad u_2=2$

Resub.: $z^2=-4 \Rightarrow z_1=2i \quad z_2=-2i$

$z^2=2 \Rightarrow z_3=\sqrt{2} \quad z_4=-\sqrt{2}$

$$L=\{\sqrt{2};-\sqrt{2};2i;-2i\}$$

3 A6 1 a) $s_3=\sum_{k=0}^{3} 9\cdot(-0{,}8)^k = 9-7{,}2+5{,}76-4{,}608=2{,}952$ (oder $\frac{369}{125}$)

2 b) $\lim_{n\to\infty} s_n = 9\cdot\frac{1}{1-q}$ falls $|q|<1$

$q=-0{,}8 \Rightarrow \lim_{n\to\infty} s_n = 9\cdot\frac{1}{1+0{,}8}=9\cdot\frac{1}{\frac{9}{5}}=9\cdot\frac{5}{9}=5$

5 A7 3 a) $f(x)=\cos(2x) \Rightarrow f(0)=1$

$f'(x)=-2\sin(2x) \Rightarrow f'(0)=0$

$f''(x)=-4\cos(2x) \Rightarrow f''(0)=-4$

$f'''(x)=8\sin(2x) \Rightarrow f'''(0)=0$

$f^{(4)}(x)=16\cos(2x) \Rightarrow f^{(4)}(0)=16$

$$p_4(x)=\frac{1}{0!}-\frac{4}{2!}x^2+\frac{16}{4!}x^4=1-2x^2+\frac{2}{3}x^4$$

2 b) $p_4(1)=1-2+\frac{2}{3}=-\frac{1}{3}$

WTR $f(1)=\cos(2)\approx-0{,}4161$ $\Big\}$ $p_4(1)-f(1)\approx 0{,}0828$

prozentuale Abweichung: $\left|\frac{p_4(1)-f(1)}{f(1)}\cdot 100\%\right| \approx 19{,}9\%$

12 VP

3 A8 $\left|\frac{a_n}{a_{n+1}}\right| = \left|\frac{(-1)^{n-1}\cdot\frac{1}{n\cdot 4^n}}{(-1)^n\cdot\frac{1}{(n+1)\cdot 4^{(n+1)}}}\right| = \left|-\frac{(n+1)\cdot 4^{n+1}}{n\cdot 4^n}\right|$

$$= \frac{(n+1)\cdot 4}{n} = \frac{4n+4}{n} = 4+\frac{4}{n}$$

$$\lim_{n\to\infty}\left|\frac{a_n}{a_{n+1}}\right| = 4 \quad \Rightarrow \quad r=4$$

6 A9 $g(x) = \sin x\cdot\cos x \quad \Rightarrow g(0) = 0$

$$g'(x) = (\cos x)^2 + \sin x\cdot(-\sin x) = (\cos x)^2 - (\sin x)^2$$

$$g'(x) = 1-(\sin x)^2-(\sin x)^2 = 1-2\cdot(\sin x)^2 \quad \Rightarrow g'(0)=1$$

$$g''(x) = -2\cdot 2\cdot\sin x\cdot\cos x = -4\sin x\cdot\cos x = -4g(x)$$

$$\Rightarrow g''(0) = -4g(0) = 0$$

$$g'''(x) = -4g'(x) \Rightarrow g'''(0) = -4g'(0) = -4$$

$$g^{(4)}(x) = -4g''(x) = 16\,g(x) \Rightarrow g^{(4)}(0) = 16\,g(0) = 0$$

$$g^{(5)}(x) = 16\cdot g'(x) \Rightarrow g^{(5)}(0) = 16\cdot g'(0) = 16$$

$\vdots$

$g^{(k)}(0) = 0$ für gerade k $(k=2l)$

$g^{(k)}(0) = (-4)^l$ für $k = 2l+1$

$$g(x) = \sum_{l=0}^{\infty}\frac{(-4)^l}{(2l+1)!}\cdot x^{2l+1}$$

Σ 42VP (25VP komplexe Zahlen ; 17VP Reihen)

Notenspiegel Klausur Nr 3 16.01.20 VKM 12 CAN

Durchschnitt: 10,1 NP 1 Schüler $\hat{=}$ 1cm

9VP