

Vertiefungskurs Mathematik 12

Lösungen: Aufgaben zu komplexen Potenzen und Wurzeln

AUFGABE 1

a) $-5 + 12i$ b) $-117 - 44i$ c) $8 - 8i$ d) $-524288 + 524288i$

AUFGABE 2

a) $e^{\frac{5}{4}\pi i}$ b) $e^{6\pi i} = 1$ c) $e^{\frac{6}{5}\pi i}$ d) $e^{\frac{1}{10}\pi i}$ e) $e^{14i} \approx e^{1,344i}$

AUFGABE 3

a) $64e^{\frac{6}{5}\pi i}$ b) $\frac{1}{128}e^{\frac{5}{4}\pi i}$ c) -512 d) $8e^{\frac{2}{5}\pi i}$ e) $625e^{2,966i}$

AUFGABE 4

a) $n = 8$ b) $n = 40$ c) $n = 10$

AUFGABE 5

a) $z_1 = e^{\frac{1}{2}\pi i}$; $z_2 = e^{\left(\frac{1}{2} + \frac{2}{3}\right)\pi i} = e^{\frac{7}{6}\pi i}$; $z_3 = e^{\left(\frac{1}{2} + 2 \cdot \frac{2}{3}\right)\pi i} = e^{\frac{11}{6}\pi i}$

b) $z_1 = e^{\frac{1}{6}\pi i}$; $z_2 = e^{\left(\frac{1}{6} + \frac{1}{2}\right)\pi i} = e^{\frac{2}{3}\pi i}$; $z_3 = e^{\left(\frac{1}{6} + 2 \cdot \frac{1}{2}\right)\pi i} = e^{\frac{7}{6}\pi i}$; $z_4 = e^{\left(\frac{1}{6} + 3 \cdot \frac{1}{2}\right)\pi i} = e^{\frac{5}{3}\pi i}$

c) $z_1 = e^{\frac{1}{12}\pi i}$; $z_2 = e^{\left(\frac{1}{12} + \frac{1}{3}\right)\pi i} = e^{\frac{5}{12}\pi i}$; $z_3 = e^{\left(\frac{1}{12} + 2 \cdot \frac{1}{3}\right)\pi i} = e^{\frac{3}{4}\pi i}$

$z_4 = e^{\left(\frac{1}{12} + 3 \cdot \frac{1}{3}\right)\pi i} = e^{\frac{13}{12}\pi i}$; $z_5 = e^{\left(\frac{1}{12} + 4 \cdot \frac{1}{3}\right)\pi i} = e^{\frac{17}{12}\pi i}$; $z_6 = e^{\left(\frac{1}{12} + 5 \cdot \frac{1}{3}\right)\pi i} = e^{\frac{7}{4}\pi i}$

d) $z_1 = e^{\frac{1}{40}\pi i}$; $z_2 = e^{\left(\frac{1}{40} + \frac{1}{4}\right)\pi i} = e^{\frac{11}{40}\pi i}$; $z_3 = e^{\left(\frac{1}{40} + 2 \cdot \frac{1}{4}\right)\pi i} = e^{\frac{21}{40}\pi i}$

$z_4 = e^{\left(\frac{1}{40} + 3 \cdot \frac{1}{4}\right)\pi i} = e^{\frac{31}{40}\pi i}$; $z_5 = e^{\left(\frac{1}{40} + 4 \cdot \frac{1}{4}\right)\pi i} = e^{\frac{41}{40}\pi i}$; ; $z_6 = e^{\left(\frac{1}{40} + 5 \cdot \frac{1}{4}\right)\pi i} = e^{\frac{51}{40}\pi i}$

$z_7 = e^{\left(\frac{1}{40} + 6 \cdot \frac{1}{4}\right)\pi i} = e^{\frac{61}{40}\pi i}$; ; $z_8 = e^{\left(\frac{1}{40} + 7 \cdot \frac{1}{4}\right)\pi i} = e^{\frac{71}{40}\pi i}$

e) $z_1 = e^{0,13i}$; $z_2 = e^{\left(0,13 + \frac{2}{9}\pi\right)i} \approx e^{0,828i}$; $z_3 = e^{\left(0,13 + 2 \cdot \frac{2}{9}\pi\right)i} \approx e^{1,526i}$

$z_4 = e^{\left(0,13 + 3 \cdot \frac{2}{9}\pi\right)i} \approx e^{2,224i}$; $z_5 = e^{\left(0,13 + 4 \cdot \frac{2}{9}\pi\right)i} \approx e^{2,923i}$

$z_6 = e^{\left(0,13 + 5 \cdot \frac{2}{9}\pi\right)i} \approx e^{3,621i}$; $z_7 = e^{\left(0,13 + 6 \cdot \frac{2}{9}\pi\right)i} \approx e^{4,319i}$

$z_8 = e^{\left(0,13 + 7 \cdot \frac{2}{9}\pi\right)i} \approx e^{5,017i}$; $z_9 = e^{\left(0,13 + 8 \cdot \frac{2}{9}\pi\right)i} \approx e^{5,715i}$

AUFGABE 6

$$\text{a) } z_1 = 2e^{\frac{1}{4}\pi i}; \quad z_2 = 2e^{\left(\frac{1}{4}+\frac{2}{3}\right)\pi i} = 2e^{\frac{11}{12}\pi i}; \quad z_3 = 2e^{\left(\frac{1}{4}+2\cdot\frac{2}{3}\right)\pi i} = 2e^{\frac{19}{12}\pi i}$$

$$\text{b) } z_1 = \frac{1}{2}e^{\frac{1}{10}\pi i}; \quad z_2 = \frac{1}{2}e^{\left(\frac{1}{10}+\frac{1}{2}\right)\pi i} = \frac{1}{2}e^{\frac{3}{5}\pi i}; \quad z_3 = \frac{1}{2}e^{\left(\frac{1}{10}+2\cdot\frac{1}{2}\right)\pi i} = \frac{1}{2}e^{\frac{11}{10}\pi i}$$
$$z_4 = \frac{1}{2}e^{\left(\frac{1}{10}+3\cdot\frac{1}{2}\right)\pi i} = \frac{1}{2}e^{\frac{8}{5}\pi i}$$

$$\text{c) } z_1 = \sqrt{2}e^{\frac{1}{12}\pi i}; \quad z_2 = \sqrt{2}e^{\left(\frac{1}{12}+\frac{1}{4}\right)\pi i} = \sqrt{2}e^{\frac{1}{3}\pi i}; \quad z_3 = \sqrt{2}e^{\left(\frac{1}{12}+2\cdot\frac{1}{4}\right)\pi i} = \sqrt{2}e^{\frac{7}{12}\pi i}$$

$$z_4 = \sqrt{2}e^{\left(\frac{1}{12}+3\cdot\frac{1}{4}\right)\pi i} = \sqrt{2}e^{\frac{5}{6}\pi i}; \quad z_5 = \sqrt{2}e^{\left(\frac{1}{12}+4\cdot\frac{1}{4}\right)\pi i} = \sqrt{2}e^{\frac{13}{12}\pi i}$$

$$z_6 = \sqrt{2}e^{\left(\frac{1}{12}+5\cdot\frac{1}{4}\right)\pi i} = \sqrt{2}e^{\frac{4}{3}\pi i}; \quad z_7 = \sqrt{2}e^{\left(\frac{1}{12}+6\cdot\frac{1}{4}\right)\pi i} = \sqrt{2}e^{\frac{19}{12}\pi i}$$

$$z_8 = \sqrt{2}e^{\left(\frac{1}{12}+7\cdot\frac{1}{4}\right)\pi i} = \sqrt{2}e^{\frac{11}{6}\pi i}$$

$$\text{d) } z_1 = 2e^{\frac{6}{25}\pi i}; \quad z_2 = 2e^{\left(\frac{6}{25}+\frac{2}{5}\right)\pi i} = 2e^{\frac{16}{25}\pi i}; \quad z_3 = 2e^{\left(\frac{6}{25}+2\cdot\frac{2}{5}\right)\pi i} = 2e^{\frac{26}{25}\pi i}$$

$$z_4 = 2e^{\left(\frac{6}{25}+3\cdot\frac{2}{5}\right)\pi i} = 2e^{\frac{36}{25}\pi i}; \quad z_5 = 2e^{\left(\frac{6}{25}+4\cdot\frac{2}{5}\right)\pi i} = 2e^{\frac{46}{25}\pi i}$$

$$\text{e) } z_1 = \sqrt{3}e^{0,3\pi i}; \quad z_2 = \sqrt{3}e^{\left(0,3+\frac{1}{3}\pi\right)i} \approx \sqrt{3}e^{1,347\pi i}; \quad z_3 = \sqrt{3}e^{\left(0,3+2\cdot\frac{1}{3}\pi\right)i} \approx \sqrt{3}e^{2,394i}$$

$$z_4 = \sqrt{3}e^{\left(0,3+3\cdot\frac{1}{3}\pi\right)i} \approx \sqrt{3}e^{3,442i}; \quad z_5 = \sqrt{3}e^{\left(0,3+4\cdot\frac{1}{3}\pi\right)i} \approx \sqrt{3}e^{4,489i}$$

$$z_6 = \sqrt{3}e^{\left(0,3+5\cdot\frac{1}{3}\pi\right)i} \approx \sqrt{3}e^{5,536i}$$